Ex.14. p.20.

(a) 4-bit code: $\underline{2}\ \underline{2}\ \underline{2}\ \underline{2}$ ← each bit has 2 choices: either 0 or 1 ⇒ There $2^4 = 16$ diffrerent codes

(b) Suppose we have n bits ⇒ $2^n = 32$ ⇒ $n = \log_2 32 = 5$.

Ex. 22 p.20

20 hard drives of which 3 are defectives.

5 are chosen randomly for test ⇒ there are $\binom{20}{5}$ ways to choose 5 hard drives randomly.

Let A = {shipment is not accepted}.

The shipment is not accepted if at least one of the hard drives is defective.

The complement event:

A' = {shipment is accepte} = {all chosen hard drives are good}.

To choose 5 good hard drives, there are $\binom{17}{5}$ different ways since there are 17 non defected hard drives.

$$\Rightarrow p(A') = \frac{\binom{17}{5}}{\binom{20}{5}}$$

$$p(A) = 1 - p(A') = 1 - \frac{\binom{17}{5}}{\binom{20}{5}}.$$

Ex. 26 p.22

(a). Each switch has 3 ways => by the multiplication principle

there are 3^6 ways.

(b). There's one way to open the door

Let A be the event : A = { open the door from the first attempt }

$$P(A) = \frac{1}{3^6}.$$

(c) $\binom{6}{2} \cdot \binom{4}{2} \binom{2}{2} = \frac{6!}{2!\,4!} \cdot \frac{4!}{2!\,2!} \cdot 1 = \frac{6 \cdot \cancel{5} \cdot \cancel{4} \cdot 3 \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot 2} = 90.$

We can choose 2 places among 6 for the up switches and we have 4 left.

Then we choose 2 from 4 f the down switches and we are left with 2 switches for the center switches.

Ex. 26 p. 42

Let Q = {the person was asked a question}
Y = {the person said yes}.

$p(Q) = 0{,}5$ (since half the subjects are asked).

$p(Y|Q) = 0{,}17$ and $p(Y|Q') = 0{,}03$ where Q' is the complement of Q.

$$p(Y) = p(Y \cap Q) + p(Y \cap Q')$$
$$= p(Y|Q) \cdot p(Q) + p(Y|Q') \cdot \underbrace{p(Q')}_{1-0{,}5=0{,}5}$$
$$= 0{,}17 \cdot 0{,}5 + 0{,}03 \cdot 0{,}5 = 0{,}1$$

$p(Y|Q) \neq p(Y)$ so they are dependent.

Ex. 32 p. 42

If A_1 and A_2 were independent then

$$p(A_1 \cap A_2) = p(A_1) \cdot p(A_2)$$

But since A_1 and A_2 are disjoint then $p(A_1 \cap A_2) = 0$

and $p(A_1) \cdot p(A_2) > 0$ Contradiction.

$\Rightarrow$ A_1 and A_2 are dependent.

Ex. 40 p. 44

Let A = {substation A experiences an overload}
B = { " B " " " }
C = { " C " " " }

$p(A) = 0{,}6$ $p(B) = 0{,}2$ and $p(C) = 0{,}15$

Scanned by CamScanner

$p(\text{Blackout} \mid A) = 0{,}01$ $\quad$ $p(\text{Blackout} \mid B) = 0{,}02$

$p(\text{Blackout} \mid C) = 0{,}03.$

$p(\text{Blackout} \mid \geq 2 \text{ overload}) = 0{,}05$ $\quad$ $p(\geq 2 \text{ overload}) = 0{,}05.$

Find $p(A \mid \text{Blackout})$, $p(B \mid \text{Blackout})$, $p(C \mid \text{Blackout})$

$p(\geq 2 \text{ overload} \mid \text{Blackout}).$

Bayes' Theorem:

$$p(\text{Blackout}) = p(\text{Blackout} \mid A) \cdot p(A) + p(\text{Black-} \mid B)\, p(B) + p(\text{Bla-} \mid C)\, p(C) + p(\text{Bla} \mid \geq 2) \cdot p(\geq 2)$$

$$= 0{,}01 \cdot 0{,}6 + 0{,}02 \cdot 0{,}2 + 0{,}03 \cdot 0{,}15 + 0{,}05 \cdot 0{,}05$$

$$= 0{,}017.$$

$$p(A \mid \text{Blackout}) = \frac{p(\text{Blackout} \mid A) \cdot p(A)}{p(\text{Blackout})} = \frac{0{,}01 \cdot 0{,}6}{0{,}017} \approx 0{,}353$$

$$p(B \mid \text{Blackout}) = \frac{p(\text{Blackout} \mid B) \cdot p(B)}{p(\text{Blackout})} = \frac{0{,}02 \cdot 0{,}2}{0{,}17} = 0{,}235$$

$$p(C \mid \text{Blackout}) = \frac{0{,}03 \cdot 0{,}15}{0{,}017} = 0{,}265$$

$$p(\geq 2 \mid \text{Blackout}) = \frac{0{,}05 \cdot 0{,}05}{0{,}017} = 0{,}147.$$