Ex. 56 p. 256 (MA)

(a) $\bar{X} = \frac{\sum x_i}{36} = 7,14$

(b) Since $n = 36$ is large enough then $\bar{X}$ is approximately normally distributed with mean μ and variance $\frac{\sigma^2}{n} = \frac{25}{36}$

$\bar{X} \sim N(\mu, \frac{25}{36})$

(c). CI $\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} ; \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$.

$\alpha = 0,02 \Rightarrow \frac{\alpha}{2} = 0,01 \Rightarrow z_{0,01} = 2,325$ (table)

$\Rightarrow$ C.I. $= \left(7,14 - 2,325 \cdot \frac{5}{6} ; 7,14 + 2,325 \cdot \frac{5}{6}\right)$

$= (5,2 ; 9,08)$

(d) Yes since 10 is outside the C.I. The probability of getting a mean greater then 10 should be less than 1%.

Ex. 64 p. 256

Let X be the result of a single toss of a die

Distribution of X:

X	1	2	3	4	6	6
p	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$\mu = E[X] = \sum x p(x) = 3,5$

$E[X^2] = \sum x^2 p(x) = 15,167$

$\Rightarrow \sigma^2 = 15,167 - 3,5^2 = 2,917.$

(a) $\bar{X} = 2,83$

a 95% : $\left(\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \; ; \; \bar{X} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \right)$.

$\frac{\alpha}{2} = 0,025 \Rightarrow z_{\frac{\alpha}{2}} = 1,96$

$\Rightarrow$ C.I. : $\left(2,83 - 1,96 \cdot \sqrt{\frac{2,917}{30}} \; ; \; 2,83 + 1,96 \sqrt{\frac{2,917}{30}} \right)$.

$= (2,22 \; ; \; 3,44)$.

The interval does not contain 3,5.

(b) a 90% C.I. is narrower than a 95% C.I.

then it won't contain 3,5.

(c) a 99% C.I is wider than a 95% C.I so it

might contain 3,5.

(d) 99% C.I.

$\alpha = 0,01 \Rightarrow \frac{\alpha}{2} = 0,005 \Rightarrow z_{\frac{\alpha}{2}} = 2,575$.

$\Rightarrow$ C.I = $(2,03 \; ; \; 3,63)$. contains the mean.

Ex.2 p. 289 (MA)

a) $\overline{X} = 8{,}62$

$$\Rightarrow \quad s^2 = \frac{\sum (x_i - \overline{x})^2}{n-1} = 20{,}43.$$

b). df = 29.

C.I.: $$\left(\frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} ; \frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}} \right)$$

$$\alpha = 0{,}1 \Rightarrow \frac{\alpha}{2} = 0{,}05 \Rightarrow \chi^2_{\frac{\alpha}{2}} = 42{,}6$$

$$\chi^2_{1-\frac{\alpha}{2}} = 17{,}7$$

$$\Rightarrow \text{C.I.}: \left(\frac{29 \cdot 20{,}43}{42{,}6} ; \frac{29 \cdot 20{,}43}{17{,}7} \right)$$

$$= (13{,}91 ; 33{,}47).$$

(c) C.I. on σ $= \left(\sqrt{13{,}90} ; \sqrt{33{,}47}\right)$

$= (3{,}73 ; 5{,}79).$

(d) $(\overline{X} - 2s ; \overline{X} + 2s) \approx (-0{,}41 ; 17{,}7)$

Since the data are positive so it's enough to take

(0; 17,7).

18 > upperbound of the interval, then it's unusual.

Ex.10 p. 291

(a) $\bar{X} = 2$ $S^2 = 0,302$ $S = \sqrt{S^2} = 0,55.$

(b) $\left(\bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}\right).$ Since σ unknown $\Rightarrow$ we use t - distribution

$\frac{\alpha}{2} = 0,05$, $df = 9$

$\Rightarrow t_{\frac{\alpha}{2}} = t_{0,05} = 1,833$

$\Rightarrow$ C.I. $= \left(2 - 1,833 \cdot \frac{0,55}{\sqrt{10}} ; \quad 2 + 1,833 \cdot \frac{0,55}{\sqrt{10}}\right)$

$= (1,68 ; \quad 2,32)$