Ex.18 (EG)

(a) $1, 5, 5^2, 5^3, 5^4, 5^5, \ldots$

$$g(x) = \sum_{n\geq 0} a_n x^n \quad \text{where} \quad a_n = 5^n, \quad n \geq 0$$

$$\Rightarrow \quad g(x) = \sum_{n\geq 0} (5x)^n = \frac{1}{1-5x} \quad \text{for} \quad |5x| < 1$$

(b) $-2, 4, -8, 16, -32, 64, \ldots$

$$g(x) = \sum_{n\geq 1} (-2)^n x^n = \sum_{n\geq 1} (-2x)^n = \frac{-2x}{1+2x} \quad \text{for} \quad |-2x| < 1$$

(c) $\binom{1/5}{0}, \binom{1/5}{1}, \binom{1/5}{2}, \ldots$

$$g(x) = \sum_{n\geq 0} \binom{1/5}{n} x^n = (1+x)^{1/5}$$

(d) $\binom{17}{17}, \binom{18}{17}, \binom{19}{17}, \binom{20}{17}, \ldots$

$$\sum_{n\geq 0} \binom{n+k}{k} x^n = \frac{1}{(1-x)^{17+1}} = \frac{1}{(1-x)^{18}}$$

Ex. 19 (EG)

$$A(x) = \sum_{n\geq 0} a_n x^n, \qquad B(x) = \sum_{n\geq 0} b_n x^n$$

(a) $$A(x) + B(x) = \sum_{n\geq 0} a_n x^n + \sum_{n\geq 0} b_n x^n = \sum_{n\geq 0} \underbrace{(a_n + b_n)}_{c_n} x^n$$

Låt $c_n = a_n + b_n \quad n \geq 0$

$A(x) + B(x)$ is the generating function for c_n.

(b) $A(x)B(x) = \left(\sum_{n\geq 0} a_n x^n\right)\left(\sum_{n\geq 0} b_n x^n\right)$

$$= (a_0 + a_1 x + a_2 x^2 + \cdots)(b_0 + b_1 x + b_2 x^2 + \cdots)$$

$$= a_0 b_0 + (a_1 b_0 + a_0 b_1)x + (a_0 b_2 + a_1 b_1 + a_2 b_0)x^2 + \cdots$$

Let $c_n = \sum_{i=0}^{n} a_i b_{n-i}$ $\leftarrow$ coeficient of x^n in $A(x)B(x)$

then $A(x)B(x)$ is the generating function for c_n.

(c) $A(x^2) = \sum_{n\geq 0} a_n (x^2)^n = \sum_{n\geq 0} a_n x^{2n}$

$$= a_0 + a_1 x^2 + a_2 x^4 + a_3 x^6 + \cdots$$

$$= a_0 + 0\cdot x + a_1 x^2 + 0\cdot x^3 + a_2 x^4 + \cdots$$

Let $c_n = \begin{cases} 0 & \text{if } n \text{ is odd} \\ a_{\frac{n}{2}} & \text{if } n \text{ is even} \end{cases}$

$A(x^2)$ is the generating function of c_n.