29.

$$(*)\begin{cases} a_0 = 1 \\ a_1 = 3 \\ a_n = 7a_{n-1} - 10a_{n-2} \qquad n>1 \end{cases}$$

$$\begin{cases} f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots \\ 7x f(x) = 7a_0 x + 7a_1 x^2 + 7a_2 x^3 + \cdots \\ -10x^2 f(x) = -10a_0 x^2 - 10a_1 x^3 + \cdots \end{cases}$$

$$(1 - 7x + 10x^2) f(x) = a_0 + (a_1 - 7a_0)x + \underbrace{(a_2 - 7a_1 + 10a_0)}_{=0} x^2 + \cdots + \underbrace{(a_n - 7a_{n-1} + 10a_{n-2})}_{=0} x^n + \cdots$$

$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for all $n>1$ $(*)$.

$$\Rightarrow (1 - 7x + 10x^2) f(x) = a_0 + (a_1 - 7a_0)x = 1 + (3-7)x = 1 - 4x$$

$$\Rightarrow f(x) = \frac{1-4x}{1-7x+10x^2}$$

$$1 - 7x + 10x^2 = (1-2x)(1-5x)$$

$$\Rightarrow f(x) = \frac{1-4x}{(1-2x)(1-5x)} = \frac{A}{1-2x} + \frac{B}{1-5x}$$

$$A = \left.\frac{1-4x}{1-5x}\right|_{x=\frac{1}{2}} = \frac{1-2}{1-\frac{5}{2}} = \frac{-1}{-3}\cdot 2 = \frac{2}{3}.$$

$$B = \left.\frac{1-4x}{1-2x}\right|_{x=\frac{1}{5}} = \frac{1-\frac{4}{5}}{1-\frac{2}{5}} = \frac{\frac{5-4}{5}}{\frac{5-2}{5}} = \frac{1}{3}$$

$$\Rightarrow \quad f(x) = \frac{2}{3} \cdot \frac{1}{1-2x} + \frac{1}{3} \cdot \frac{1}{1-5x}$$

$$= \frac{2}{3} \sum_{n \geq 0} (2x)^n + \frac{1}{3} \sum_{n \geq 0} (5x)^n$$

$$= \sum_{n \geq 0} \left(\frac{2}{3} \cdot 2^n + \frac{1}{3} \cdot 5^n \right) x^n$$

$$= \sum_{n \geq 0} \left(\frac{2^{n+1}}{3} + \frac{5^n}{3} \right) x^n$$

$$\Rightarrow a_n = \frac{2^{n+1}}{3} + \frac{5^n}{3} .$$