Financial Time Series – Introduction to time series

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What is a time series?

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Definition

A *time series* is a real-valued sequence of observations $(x_t, t \in \mathbb{T})$ with respect to an index set $\mathbb{T} \subset \mathbb{R}$. A *time series model* for the observed data $(x_t, t \in \mathbb{T})$ is a specification of the joint distributions (or possibly only the means and covariances) of a sequence of random variables $(X_t, t \in \mathbb{T})$ of which $(x_t, t \in \mathbb{T})$ postulates to be a realization.

What is a time series?

- Focus on *discrete time series*: Assume that $\mathbb{T}$ is a discrete set
- $\mathbb{T} = \{t_n, n \in \mathbb{N}\}$
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- Examples:

Quarterly earnings of H&M

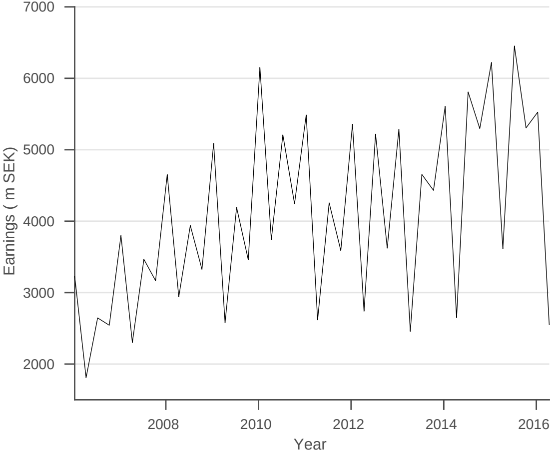

Figure: Quarterly earnings of H&M from January 2006 through April 2016.

Log-returns

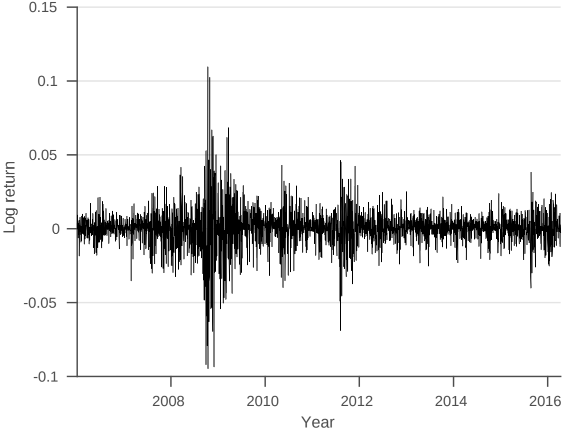

Figure: Daily log-returns of the S&P500 index from January 2006 to April 2016.

Notation and distribution for discrete time series

- For $\{t_n, n \in \mathbb{N}\}$, abbreviate $(X_{t_n}, n \in \mathbb{N})$ by $(X_n, n \in \mathbb{N})$ or $(X_n, n \in \mathbb{Z})$
- Similar for observations: $(x_n, n \in \mathbb{N})$
- For finite observations and models: $(x_1, \ldots, x_n)$ and $(X_1, \ldots, X_n)$

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Specification of the joint distributions = knowledge of all probabilities

$$P_{X_{t_1},\ldots,X_{t_m}}((-\infty, y_1], \ldots, (-\infty, y_m]) = P(X_{t_1} \leq y_1, \ldots, X_{t_m} \leq y_m)$$

for all finite random vectors $(X_{t_1}, \ldots, X_{t_m})$ of any $\{t_1, \ldots, t_m\} \subset \mathbb{N}$ with finite $m \in \mathbb{N}$ and all $y_j \in \mathbb{R}$, $j = 1, \ldots, m$.

IID noise

Definition

A stochastic process $X = (X_t, t \in \mathbb{T})$ is called *iid noise* with mean μ and variance σ^2 if the sequence of random variables $(X_t, t \in \mathbb{T})$ is independent and identically distributed (abbreviated by *iid*) with $\mathbb{E}(X_t) = \mu$ and $\mathrm{Var}(X_t) = \sigma^2$ for all $t \in \mathbb{T}$.

Binary process

- Example of IID noise: flipping a fair coin

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- $(X_n, n \in \mathbb{Z})$ sequence of IID random variables characterized by:

Random walk

- Given iid noise $(X_n, n \in \mathbb{N})$, a random walk $(S_n, n \in \mathbb{N}_0)$ is obtained by the cumulative summing of X

Gaussian time series

Definition

A time series X is said to be a *Gaussian time series* if all finite-dimensional distributions are normal, i.e., all finite-dimensional vectors are multivariate Gaussian distributed.